Subject: A System for Typesetting Mathematics

Case- -- File-

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TM:

MEMORANDUM FOR FILE

1. Introduction

"Mathematics is known in the trade as *difficult*, or *penalty, copy* because it is slower, more difficult, and more expensive to set in type than any other kind of copy normally occurring in books and journals." [1] One difficulty with mathematical text is the multiplic-

ity of characters, sizes, and fonts. An expression such as

$$\lim_{x \to \pi/2} (\tan x)^{\sin 2x} = 1$$

requires an intimate mixture of roman, italic and greek letters, in three sizes, and a special character or two. ("Requires" is perhaps the wrong word, but mathematics has its own typographical conventions which are quite different from those of ordinary text.) Typesetting such an expression by traditional methods is still an essentially manual operation.

A second difficulty is the two dimensional character of mathematics, which the superscript and limits in the preceding example showed in its simplest form. This is carried further by

$$a_0 + \frac{b_1}{a_1 + \frac{b_2}{a_2 + \frac{b_3}{a_3 + \cdots}}}$$

and still further by

$$\int \frac{dx}{ae^{mx} - be^{-mx}} = \begin{cases} \frac{1}{2m\sqrt{ab}} \log \frac{\sqrt{a}e^{mx} - \sqrt{b}}{\sqrt{a}e^{mx} + \sqrt{b}} \\ \frac{1}{m\sqrt{ab}} \tanh^{-1}(\frac{\sqrt{a}}{\sqrt{b}}e^{mx}) \\ \frac{-1}{m\sqrt{ab}} \coth^{-1}(\frac{\sqrt{a}}{\sqrt{b}}e^{mx}) \end{cases}$$

These examples also show line-drawing, built-up characters like braces and radicals, and a spectrum of positioning problems. (Section 6 shows what a user has to type to produce these on our system.)

2. Photocomposition

Photocomposition techniques can be used to solve some of the problems of typesetting mathematics. A phototypesetter is a device which exposes a piece of photographic paper or film, placing characters wherever they are wanted. The Graphic Systems phototypesetter[2] on the UNIX operating system[3] works by shining light through a character stencil. The character is made the right size by lenses, and the light beam directed by fiber optics to the desired place on a piece of photographic paper. The exposed paper is developed and typically used in some form of photo-offset reproduction.

On UNIX, the phototypesetter is driven by a formatting program called TROFF [4]. TROFF was designed for setting running text. It also provides all of the facilities that one needs for doing mathematics, such as arbitrary horizontal and vertical motions, line-drawing, size changing, but the syntax for describing these special operations is difficult to learn, and difficult even for experienced users to type correctly.

For this reason we decided to use TROFF as an "assembly language," by designing a language for describing mathematical expressions, and compiling it into TROFF.

3. Language Design

The fundamental principle upon which we based our language design is that the language should be easy to use by people (for example, secretaries) who know neither mathematics nor typesetting.

This principle implies several things. First, "normal" mathematical conventions about operator precedence, parentheses, and the like cannot be used, for to give special meaning to such characters means that the user has to understand what he or she is typing. Thus the language should not assume, for instance, that parentheses are always balanced, for they are not in the half-open interval (a, b]. Nor should it assume that that $\sqrt{a+b}$ can be replaced by $(a+b)^{\frac{1}{2}}$, or that 1/(1-x) is better written as $\frac{1}{1-x}$ (or vice versa). Second, there should be relatively few rules, keywords,

Second, there should be relatively few rules, keywords, special symbols and operators, and the like. This keeps the language easy to learn and remember. Furthermore, there should be few exceptions to the rules that do exist: if something works in one situation, it should work everywhere. If a variable can have a subscript, then a subscript can have a subscript, and so on without limit.

Third, "standard" things should happen automatically. Someone who types "x=y+z+1" should get "x = y + z + 1". Subscripts and superscripts should automatically be printed in an appropriately smaller size, with no special intervention. Fraction bars have to be made the right length and positioned at the right height. And so on. Indeed a mechanism for overriding default actions has to exist, but its application is the exception, not the rule.

We assume that the typist has a reasonable picture (a two-dimensional representation) of the desired final form, as might be handwritten by the author of a paper. We also assume that the input is typed on a computer terminal much like an ordinary typewriter. This implies an input alphabet of perhaps 100 characters, none of them special.

A secondary, but still important, goal in our design was that the system should be easy to implement, since neither of the authors had any desire to make a long-term project of it. Since our design was not firm, it was also necessary that the program be easy to change at any time.

To make the program easy to build and to change, and to guarantee regularity ("it should work everywhere"), the language is defined by a context-free grammar, described in Section 5. The compiler for the language was built using a compiler-compiler.

A priori, the grammar/compiler-compiler approach seemed the right thing to do. Our subsequent experience leads us to believe that any other course would have been folly. The original language was designed in a few days. Construction of a working system sufficient to try significant examples required perhaps a person-month. Since then, we have spent a modest amount of additional time over several years tuning, adding facilities, and occasionally changing the language as users make criticisms and suggestions.

We also decided quite early that we would let TROFF do our work for us whenever possible. TROFF is quite a powerful program, with a macro facility, text and arithmetic variables, numerical computation and testing, and conditional branching. Thus we have been able to avoid writing a lot of mundane but tricky software. For example, we store no text strings, but simply pass them on to TROFF. Thus we avoid having to write a storage management package. Furthermore, we have been able to isolate ourselves from most details of the particular device and character set currently in use. For example, we let TROFF compute the widths of all strings of characters; we need know nothing about them.

A third design goal is special to our environment. Since our program is only useful for typesetting mathematics, it is necessary that it interface cleanly with the underlying typesetting language for the benefit of users who want to set intermingled mathematics and text (the usual case). The standard mode of operation is that when a document is typed, mathematical expressions are input as part of the text, but marked by user settable delimiters. The program reads this input and treats as comments those things which are not mathematics, simply passing them through untouched. At the same time it converts the mathematical input into the necessary TROFF commands. The resulting ioutput is passed directly to TROFF where the comments and the mathematical parts both become text and/or TROFF commands.

4. The Language

We will not try to describe the language precisely here; interested readers may refer to the appendix for more details. Throughout this section, we will write expressions exactly as they are handed to the typesetting program (hereinafter called "EQN"), except that we won't show the delimiters that the user types to mark the beginning and end of the expression. The interface between EQN and TROFF is described at the end of this section.

As we said, typing x=y+z+1 should produce x = y + z + 1, and indeed it does. Variables are made italic, operators and digits become roman, and normal spacings between letters and operators are altered slightly to give a more pleasing appearance.

Input is free-form. Spaces and new lines in the input are used by EQN to separate pieces of the input; they are not used to create space in the output. Thus

$$\begin{array}{rcl} x &=& y \\ &+ z + 1 \end{array}$$

also gives x = y + z + 1. Free-form input is easier to type initially; subsequent editing is also easier, for an expression may be typed as many short lines.

Extra white space can be forced into the output by several characters of various sizes. A tilde "~" gives a space equal to the normal word spacing in text; a circumflex gives half this much, and a tab charcter spaces to the next tab stop.

Spaces (or tildes, etc.) also serve to delimit pieces of the input. For example, to get

$$f(t) = 2\pi \int \sin(\omega t) dt$$

we write

f(t) = 2 pi int sin (omega t)dt

Here spaces are *necessary* in the input to indicate that *sin, pi, int,* and *omega* are special, and potentially worth special treatment. EQN looks up each such string of characters in a table, and if appropriate gives it a translation. In this case, *pi* and *omega* become their greek equivalents, *int* becomes the integral sign (which must be moved down and enlarged so it looks "right"), and *sin* is made roman, following conventional mathematical practice. Parentheses, digits and operators are automatically made roman wherever found.

Fractions are specified with the keyword over:

$$a+b$$
 over $c+d+e = 1$

produces

$$\frac{a+b}{c+d+e} = 1$$

Similarly, subscripts and superscripts are introduced by the keywords *sub* and *sup*:

$$x^2 + y^2 = z^2$$

is produced by

$$x \sup 2 + y \sup 2 = z \sup 2$$

The spaces after the 2's are necessary to mark the end of the superscripts; similarly the keyword *sup* has to be marked off by spaces or some equivalent delimiter. The return to the proper baseline is automatic. Multiple levels of subscripts or superscripts are of course allowed: "x sup y sup z" is x^{y^z} . The construct "something *sub* something *sup* something" is recognized as a special case, so "x sub i sup 2" is x_i^2 instead of x_i^2 .

More complicated expressions can now be formed with these primitives:

$$\frac{\partial^2 f}{\partial x^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

is produced by

{partial sup 2 f} over {partial x sup 2} = x sup 2 over a sup 2 + y sup 2 over b sup 2

Braces $\{\}$ are used to group objects together; in this case they indicate unambiguously what goes over what on the left-hand side of the expression. The language defines the precedence of *sup* to be higher than that of *over*, so no braces are needed to get the correct association on the right side. Braces can always be used when in doubt about precedence.

The braces convention is an example of the power of using a recursive grammar to define the language. It is part of the language that if a construct can appear in some context, then *any expression* in braces can also occur in that context.

There is a *sqrt* operator for making square roots of the appropriate size: "sqrt a+b" produces $\sqrt{a+b}$, and

$$\mathbf{x} = \{-\mathbf{b} + -\operatorname{sqrt}\{\mathbf{b} \sup 2 - 4\mathbf{ac}\}\} \text{ over } 2\mathbf{a}$$

is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Since large radicals look poor on our typesetter, *sqrt* is not useful for tall expressions.

Limits on summations, integrals and similar constructions are specified with the keywords *from* and *to*. To get

$$\sum_{i=0}^{\infty} x_i \to 0$$

we need only type

sum from i=0 to inf x sub i
$$\rightarrow 0$$

Centering and making the Σ big enough and the limits smaller are all automatic. The *from* and *to* parts are both optional, and the central part (e.g., the Σ) can in fact be anything:

 $\lim \text{ from } \{x \rightarrow pi/2\} (\tan x) = \inf$

is

- 3 -

$$\lim_{x \to \pi/2} (\tan x) = \infty$$

Again, the braces indicate just what goes into the *from* part.

There is a facility for making braces, brackets, parentheses, and vertical bars of the right height, using the keywords *left* and *right*:

makes

$$\left[\frac{x+y}{2a}\right] = 1$$

A *left* need not have a corresponding *right*, as we shall see in the next example. Any characters may follow *left* and *right*, but generally only various parentheses and bars are meaningful.

Big brackets, etc., are often used with another facility, called *piles*, which make vertical piles of objects. For example, to get

$$sign(x) \equiv \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$$

we can type

The construction "left {" makes a left brace big enough to enclose the "rpile {...}", which is a rightjustified pile of "above ... above ...". "lpile" makes a left-justified pile. There are also centered piles. Because of the recursive language definition, a pile can contain any number of elements; any element of a pile can of course contain piles.

Although EQN makes a valiant attempt to use the right sizes and fonts, there are times when the default assumptions are simply not what is wanted. For instance the italic *sign* in the previous example would conventionally be in roman. Slides and transparencies often require larger characters than normal text. Thus we also provide size and font changing commands: "size 12 bold $\{A^{\tilde{x}x}=\tilde{y}\}$ " will produce A x = y. *Size* is followed by a number representing a character size in points. (One point is 1/72 inch; this paper is set in 9 point type.)

If necessary, an input string can be quoted in "...", which turns off grammatical significance, and any font or spacing changes that might otherwise be done on it. Thus we can say

$\lim roman$ "sup" x sub n = 0

to ensure that the supremum doesn't become a superscript:

$\limsup x_n = 0$

Diacritical marks, long a problem in traditional typesetting, are straightforward:

$$\underline{\dot{x}} + \hat{x} + \tilde{y} + \hat{X} + \ddot{Y} = \overline{z + Z}$$

is made by typing

x dot under + x hat + y tilde+ X hat + Y dotdot = z+Z bar

There are also facilities for globally changing default sizes and fonts, for example for making viewgraphs or for setting chemical equations. The language allows for matrices, and for lining up equations at the same horizontal position.

Finally, there is a definition facility, so a user can say

define name " ... "

at any time in the document; henceforth, any occurrence of the token "name" in an expression will be expanded into whatever was inside the double quotes in its definition. This lets users tailor the language to their own specifications, for it is quite possible to redefine keywords like *sup* or *over*. Section 6 shows an example of definitions.

The EQN preprocessor reads intermixed text and equations, and passes its output to TROFF. Since TROFF uses lines beginning with a period as control words (e.g., ".ce" means "center the next output line"), EQN uses the sequence ".EQ" to mark the beginning of an equation and ".EN" to mark the end. The ".EQ" and ".EN" are passed through to TROFF untouched, so they can also be used by a knowledgeable user to center equations, number them automatically, etc. By default, however, ".EQ" and ".EN" are simply ignored by TROFF, so by default equations are printed in-line.

".EQ" and ".EN" can be supplemented by TROFF commands as desired; for example, a centered display equation can be produced with the input:

Since it is tedious to type ".EQ" and ".EN" around very short expressions (single letters, for instance), the user can also define two characters to serve as the left and right delimiters of expressions. These characters are recognized anywhere in subsequent text. For example if the left and right delimiters have both been set to "#", the input:

Let #x sub i#, #y# and #alpha# be positive

produces:

Let x_i , y and α be positive

Running a preprocessor is strikingly easy on UNIX. To typeset text stored in file "f", one issues the command:

eqn f | troff

The vertical bar connects the output of one process (EQN) to the input of another (TROFF).

5. Language Theory

The basic structure of the language is not a particularly original one. Equations are pictured as a set of "boxes," pieced together in various ways. For example, something with a subscript is just a box followed by another box moved downward and shrunk by an appropriate amount. A fraction is just a box centered above another box, at the right altitude, with a line of correct length drawn between them.

The grammar for the language is shown below. For purposes of exposition, we have collapsed some productions. In the original grammar, there are about 70 productions, but many of these are simple ones used only to guarantee that some keyword is recognized early enough in the parsing process. Symbols in capital letters are terminal symbols; lower case symbols are non-terminals, i.e., syntactic categories. The vertical bar | indicates an alternative; the brackets [] indicate optional material. A TEXT is a string of non-blank characters or any string inside double quotes; the other terminal symbols represent literal occurrences of the corresponding keyword.

eqn : box | eqn box

box	: text
	{ eqn }
	box OVER box
	SQRT box
	box SUB box box SUP box
	$ [L C R]$ PILE { list }
	LEFT text eqn [RIGHT text]
	box [FROM box] [TO box]
	SIZE text box
	[ROMAN BOLD ITALIC] box
	box [HAT BAR DOT DOTDOT TILDE]
	DEFINE text text

list : eqn | list ABOVE eqn

The grammar makes it obvious why there are few exceptions. For example, the observation that something can be replaced by a more complicated something in braces is implicit in the productions:

eqn : box | eqn box box : text | { eqn }

Anywhere a single character could be used, *any* legal construction can be used.

Clearly, our grammar is highly ambiguous. What, for instance, do we do with the input

a over b over c ?

Is it

```
{a over b} over c
```

or is it

a over {b over c} ?

To answer questions like this, the grammar is supplemented with a small set of rules that describe the precedence and associativity of operators. In particular, we specify (more or less arbitrarily) that over associates to the left, so the first alternative above is the one chosen. On the other hand, sub and sup bind to the right, because this is closer to standard mathematical practice. That is, we assume x^{a^b} is $x^{(a^b)}$, not $(x^a)^b$.

The precedence rules resolve the ambiguity in a construction like

a sup 2 over b

We define *sup* to have a higher precedence than *over*, so this construction is parsed as $\frac{a^2}{b}$ instead of $a^{\frac{2}{b}}$. Naturally, a user can always force a particular parsing

by placing braces around expressions.

The ambiguous grammar approach seems to be quite useful. The grammar we use is small enough to be easily understood, for it contains none of the productions that would be normally used for resolving ambiguity. Instead the supplemental information about precedence and associativity (also small enough to be understood) provides the compiler-compiler with the information it needs to make a fast, deterministic parser for the specific language we want. When the language is supplemented by the disambiguating rules, it is in fact LR(1) and thus easy to parse[5].

The output code is generated as the input is scanned. Any time a production of the grammar is recognized, (potentially) some TROFF commands are output. For example, when the lexical analyzer reports that it has found a TEXT (i.e., a string of contiguous characters), we have recognized the production:

text : TEXT

The translation of this is simple. We generate a local name for the string, then hand the name and the string to TROFF, and let TROFF perform the storage management. All we save is the name of the string, its height, and its baseline.

As another example, the translation associated with the production

box : box OVER box

is:

Width of output box =

slightly more than largest input width Height of output box =

slightly more than sum of input heights

Base of output box =

slightly more than height of bottom input box String describing output box =

move down; move right enough to center bottom box; draw bottom box (i.e., copy string for bottom box); move up; move left enough to center top box; draw top box (i.e., copy string for top box); move down and left; draw line full width; return to proper base line.

Most of the other productions have equally simple semantic actions. Picturing the output as a set of properly placed boxes makes the right sequence of positioning commands quite obvious. The main difficulty is in finding the right numbers to use for esthetically pleasing positioning.

With a grammar, it is usually clear how to extend the language. For instance, one of our users suggested a TENSOR operator, to make constructions like

${}^{l}_{m}\mathbf{T}^{k}_{\mathbf{T}}$

Grammatically, this is easy: it is sufficient to add a production like

box : TENSOR { list }

Semantically, we need only juggle the boxes to the right places.

6. Experience

There are really three aspects of interest-how well EQN sets mathematics, how well it satisfies its goal of being "easy to use," and how easy it was to build.

The first question is easily addressed. This entire paper has been set by the program. Readers can judge for themselves whether it is good enough for their purposes. One of our users commented that although the output is not as good as the best hand-set material, it is still better than average, and much better than the worst. In any case, who cares? Printed books cannot compete with the birds and flowers of illuminated manuscripts on esthetic grounds, either, but they have some clear economic advantages.

Some of the deficiencies in the output could be cleaned up with more work on our part. For example, we sometimes leave too much space between a roman letter and an italic one. If we were willing to keep track of the fonts involved, we could do this better more of the time.

Some other weaknesses are inherent in our output device. It is hard, for instance, to draw a line of an arbitrary length without getting a perceptible overstrike at one end.

As to ease of use, at the time of writing, the system has been used by two distinct groups. One user population consists of mathematicians, chemists, physicists, and computer scientists. Their typical reaction has been something like:

(1)It's easy to write, although I make the following mistakes...

(2)How do I do...?

(3)It botches the following things.... Why don't you fix them?

(4)You really need the following features...

The learning time is short. A few minutes gives the general flavor, and typing a page or two of a paper generally uncovers most of the misconceptions about how it works.

The second user group is much larger, the secretaries and mathematical typists who were the original target of the system. They tend to be enthusiastic converts. They find the language easy to learn (most are largely self-taught), and have little trouble producing the output they want. They are of course less critical of the esthetics of their output than users trained in mathematics. After a transition period, most find using a computer more interesting than a regular typewriter.

The main difficulty that users have seems to be remembering that a blank is a delimiter; even experienced users use blanks where they shouldn't and omit them when they are needed. A common instance is typing

f(x sub i)

which produces

$f(x_i)$

instead of

$$f(x_i)$$

Since the EQN language knows no mathematics, it cannot deduce that the right parenthesis is not part of the subscript.

The language is somewhat prolix, but this doesn't seem excessive considering how much is being done, and it is certainly more compact than the corresponding TROFF commands. For example, here is the source for the continued fraction expression in Section 1 of this paper:

```
a sub 0 + b sub 1 over
{a sub 1 + b sub 2 over
{a sub 2 + b sub 3 over
{a sub 3 + ... }}}
```

This is the input for the large integral of Section 1; notice the use of definitions:

As to ease of construction, we have already mentioned that there are really only a few person-months invested. Much of this time has gone into two things—fine-tuning (what is the most esthetically pleasing space to use between the numerator and denominator of a fraction?), and changing things found deficient by our users (shouldn't a tilde be a delimiter?).

The program consists of a number of small, essentially unconnected modules for code generation, a simple lexical analyzer, a canned parser which we did not have to write, and some miscellany associated with input files and the macro facility. The program is now about 1600 lines of C [6], a high-level language reminiscent of BCPL. About 20 percent of these lines are "print" statements, generating the output code.

The semantic routines that generate the actual TROFF commands can be changed to accommodate other formatting languages and devices. For example, in less than 24 hours, one of us changed the entire semantic package to drive NROFF, a variant of TROFF, for typesetting mathematics on teletypewriter devices capable of reverse line motions. Since many potential users do not have access to a typesetter, but still have to type mathematics, this provides a way to get a typed version of the final output which is close enough for debugging purposes, and sometimes even for ultimate use.

7. Conclusions

We think we have shown that it is possible to do acceptably good typesetting of mathematics on a phototypesetter, with an input language that is easy to learn and use and that satisfies many users' demands. Such a package can be implemented in short order, given a compiler-compiler and a decent typesetting program underneath.

Defining a language, and building a compiler for it with a compiler-compiler seems like the only sensible way to do business. Our experience with the use of a grammar and a compiler-compiler has been uniformly favorable. If we had written everything into code directly, we would have been locked into our original design. Furthermore, we would have never been sure where the exceptions and special cases were. But because we have a grammar, we can change our minds readily and still be reasonably sure that if a construction works in one place it will work everywhere.

Acknowledgements

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Typesetting Mathematics — User's Guide (Second Edition)

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ABSTRACT

This is the user's guide for a system for typesetting mathematics, using the phototypeset-

and GCOS operating systems.

Mathematical expressions are described in a language designedings bode asy on yourse beo The whosk now monthly a mathematics nor typesetting. Enough of the language to set in-line expressions like 1the (EROFFin and NBOEFsphacequariokagekpackage

$$G(z) = e^{\ln G(z)} = \exp\left(\sum_{k\geq 1} \frac{S_k z^k}{k}\right) = \prod_{k\geq 1} e^{S_k z^{k}/k}$$
$$= \left(1 + S_1 z + \frac{S_1^2 z^2}{2!} + \cdots\right) \left(1 + \frac{S_2 z^2}{2} + \frac{S_2^2 z^4}{2^2 \cdot 2!} + \frac{S_2^2 z^4}{2!} + \frac{S_2^$$

†UNIX is a Trademark of Bell Laboratories.

can be learned in an hour or so.

The language interfaces directly with the phototypesetting language TROFF, so the running text of a manuscript, and the entire document produced in one proce The same language may be used with the UNIX formatter NROFF to set mathema Model 37 teletypes.

EQN is a program for typesetting mathematics on the Graphics Systems phototypesetters on UNIX and GCOS. The EQN language was designed to be easy to use by people who know neither mathematics nor typesetting. Thus EQN knows relatively little about mathematics. In particular, mathematical symbols like +, -, ×, parentheses, and so on have no special meanings. EQN is quite happy to set garbage (but it will look good).

EQN works as a preprocessor for the typesetter formatter, TROFF[1], so the normal mode of operation is to prepare a document with both mathematics and ordinary text interspersed, and let EQN set the mathematics while TROFF does the body of the text.

On UNIX, EQN will also produce mathematics on DASI and GSI terminals and on Model 37 teletypes. The input is identical, but you have to use the programs NEQN and NROFF instead of EQN and TROFF. Of course, some things won't look as good because terminals don't provide the variety of characters, sizes and fonts that a typesetter does, but the output is usually adequate for proofreading. To use EQN on UNIX,

eqn files | troff

GCOS use is discussed in section 26.

To tell EQN where a mathematical expression begins and ends, we mark it with lines beginning .EQ and .EN. Thus if you type the lines

$$\neg EQ$$

x=y+z
 $\neg EN$

your output will look like

x = y + z

The .EQ and .EN are copied through untouched; they are not otherwise processed by EQN. This means that you have to take care of things like centering, number-

 $-ms^{\pi}$ developed by M. E. Lesk[3], which allows you to center, indent, left-justify and number equations. With the '-ms' package, equations are centered by default. To left-justify an equation, use .EQ L instead of .EQ. . To indent it, use .EQ I. Any of these can be followed by an arbitrary 'equation number' which will be placed at the right margin. For example, the input

$$\overset{m}{\neg} EQ I (3.1a)$$
$$x = f(y/2) + y/2$$
$$\neg EN$$

produces the output

$$x = f(y/2) + y/2$$
(3.1a)

There is also a shorthand notation so in-line expressions like π_i^2 can be entered without .EQ and .EN. We will talk about it in section 19.

Spaces and newlines within an expression are thrown away by EQN. (Normal text is left absolutely alone.) Thus between .EQ and .EN,

x=y+z

and

 $\mathbf{x} = \mathbf{y} + \mathbf{z}$

and

$$x = y$$

+ z

and so on all produce the same output

x = y + z

You should use spaces and newlines freely to make your input equations readable and easy to edit. In particular, very long lines are a bad idea, since they are often hard to fix if you make a mistake.

To force extra spaces into the *output*, use a tilde "~" for each space you want:

x~=~y~+~z

gives

$$x = y + z$$

You can also use a circumflex "^", which gives a space half the width of a tilde. It is mainly useful for finetuning. Tabs may also be used to position pieces of an expression, but the tab stops must be set by TROFF commands.

EQN knows some mathematical symbols, some mathematical names, and the Greek alphabet. For example,

x=2 pi int sin (omega t)dt

produces

$$x = 2\pi \int \sin(\omega t) dt$$

Here the spaces in the input are **necessary** to tell EQN that *int, pi, sin* and *omega* are separate entities that should get special treatment. The *sin,* digit 2, and parentheses are set in roman type instead of italic; *pi* and *omega* are made Greek; and *int* becomes the integral sign.

When in doubt, leave spaces around separate parts of the input. A *very* common error is to type f(pi) without leaving spaces on both sides of the *pi*. As a result, EQN does not recognize *pi* as a special word, and it appears as f(pi) instead of $f(\pi)$.

A complete list of EQN names appears in section 23. Knowledgeable users can also use TROFF four-character names for anything EQN doesn't know about, like (bs for the Bell System sign.

The only way EQN can deduce that some sequence of letters might be special is if that sequence is separated from the letters on either side of it. This can be done by surrounding a special word by ordinary spaces (or tabs or newlines), as we did in the previous section. You can also make special words stand out by sur-

x~=~2~pi~int~sin~(~omega~t~)~dt

rounding them with tildes or circumflexes:

is much the same as the last example, except that the tildes not only separate the magic words like *sin*, *omega*, and so on, but also add extra spaces, one space per tilde:

$$x = 2 \pi \int \sin(\omega t) dt$$

Special words can also be separated by braces { } and double quotes "...", which have special meanings that we will see soon.

Subscripts and superscripts are obtained with the words *sub* and *sup*.

 $x \sup 2 + y \sup k$

gives

$$x^{2} + y_{k}$$

EQN takes care of all the size changes and vertical motions needed to make the output look right. The words *sub* and *sup* must be surrounded by spaces; *x sub2* will give you *xsub2* instead of x_2 . Furthermore, don't forget to leave a space (or a tilde, etc.) to mark the end of a subscript or superscript. A common error is to say something like

 $y = (x \sup 2) + 1$

which causes

 $y = (x^{2)+1}$

instead of the intended

$$v = (x^2) + 1$$

Subscripted subscripts and superscripted superscripts also work:

x sub i sub 1

is

is

 x_{i_1}

A subscript and superscript on the same thing are printed one above the other if the subscript comes *first*:

x sub i sup 2

 x_i^2

Other than this special case, *sub* and *sup* group to the right, so *x sup y sub z* means x^{y_z} , not x^{y_z} .

Normally, the end of a subscript or superscript is marked simply by a blank (or tab or tilde, etc.) What if the subscript or superscript is something that has to be typed with blanks in it? In that case, you can use the braces { and } to mark the beginning and end of the subscript or superscript: - 9 -

e sup {i omega t}

is

 $e^{i\omega t}$

Rule: Braces can *always* be used to force EQN to treat something as a unit, or just to make your intent perfectly clear. Thus:

x sub {i sub 1} sup 2

is

 $x_{i_1}^2$

with braces, but

x sub i sub 1 sup 2

is

$$x_{i_{1}^{2}}$$

which is rather different. Braces can occur within braces if necessary:

e sup {i pi sup {rho +1}}

is

 $e^{i\pi^{\rho+1}}$

The general rule is that anywhere you could use some single thing like x, you can use an arbitrarily complicated thing if you enclose it in braces. EQN will look after all the details of positioning it and making it the right size.

In all cases, make sure you have the right number of braces. Leaving one out or adding an extra will cause EQN to complain bitterly.

Occasionally you will have to print braces. To do this, enclose them in double quotes, like "{". Quoting is discussed in more detail in section 14.

To make a fraction, use the word over:

a+b over 2c =1

gives

$$\frac{a+b}{2c} = 1$$

The line is made the right length and positioned automatically. Braces can be used to make clear what goes over what:

 $\{alpha + beta\}$ over $\{sin(x)\}$

is

$$\frac{\alpha + \beta}{\sin(x)}$$

What happens when there is both an *over* and a *sup* in the same expression? In such an apparently ambiguous case, EQN does the *sup* before the *over*; so

-b sup 2 over pi

is $\frac{-b^2}{\pi}$ instead of $-b^{\frac{2}{\pi}}$ The rules which decide which

operation is done first in cases like this are summarized in section 23. When in doubt, however, *use braces* to make clear what goes with what. To draw a square root, use *sqrt*:

sqrt
$$a+b+1$$
 over sqrt {ax sup $2+bx+c$ }

is

$$\sqrt{a+b} + \frac{1}{\sqrt{ax^2 + bx + c}}$$

Warning — square roots of tall quantities look lousy, because a root-sign big enough to cover the quantity is too dark and heavy:

is

$$\sqrt{\frac{a^2}{b_2}}$$

Big square roots are generally better written as something to the power $\frac{1}{2}$:

 $(a^2/b_2)^{\frac{1}{2}}$

which is

(a sup 2 /b sub 2) sup half

Summations, integrals, and similar constructions are easy:

sum from i=0 to {i= inf} x sup i

produces

$$\sum_{i=0}^{i=\infty} x^i$$

Notice that we used braces to indicate where the upper part $i = \infty$ begins and ends. No braces were necessary for the lower part i = 0, because it contained no blanks. The braces will never hurt, and if the *from* and *to* parts contain any blanks, you must use braces around them.

The *from* and *to* parts are both optional, but if both are used, they have to occur in that order.

Other useful characters can replace the *sum* in our example:

int prod union inter

become, respectively,

Since the thing before the *from* can be anything, even something in braces, *from-to* can often be used in unexpected ways:

 $\lim \text{ from } \{n \rightarrow \inf\} x \text{ sub } n = 0$

is

$$\lim x_n = 0$$

By default, equations are set in 10-point type (the same size as this guide), with standard mathematical conven-

tions to determine what characters are in roman and what in italic. Although EQN makes a valiant attempt to use esthetically pleasing sizes and fonts, it is not perfect. To change sizes and fonts, use *size n* and *roman*, *italic*, *bold* and *fat*. Like *sub* and *sup*, size and font changes affect only the thing that follows them, and revert to the normal situation at the end of it. Thus

bold x y

is

хy

and

```
size 14 bold x = y +
size 14 {alpha + beta}
```

gives

$$\mathbf{X} = y + \alpha + \beta$$

As always, you can use braces if you want to affect something more complicated than a single letter. For example, you can change the size of an entire equation by

size 12 { ... }

Legal sizes which may follow *size* are 6, 7, 8, 9, 10, 11, 12, 14, 16, 18, 20, 22, 24, 28, 36. You can also change the size *by* a given amount; for example, you can say *size* +2 to make the size two points bigger, or *size* -3 to make it three points smaller. This has the advantage that you don't have to know what the current size is.

If you are using fonts other than roman, italic and bold, you can say *font X* where *X* is a one character TROFF name or number for the font. Since EQN is tuned for roman, italic and bold, other fonts may not give quite as good an appearance.

The *fat* operation takes the current font and widens it by overstriking: *fat grad* is ∇ and *fat {x sub i}* is x_i .

If an entire document is to be in a non-standard size or font, it is a severe nuisance to have to write out a size and font change for each equation. Accordingly, you can set a "global" size or font which thereafter affects all equations. At the beginning of any equation, you might say, for instance,

¬EQ gsize 16 gfont R ... ¬EN

to set the size to 16 and the font to roman thereafter. In place of R, you can use any of the TROFF font names. The size after *gsize* can be a relative change with + or -.

often as needed. For example, in a footnote‡

the size of the footnote text, which is two points smaller than the main text. Don't forget to reset the global size at the end of the footnote.

To get funny marks on top of letters, there are several words:

x dot	ż
x dotdot	ÿ
x hat	â
x tilde	ñ
x vec	\vec{x}
x dyad	$\stackrel{\leftrightarrow}{x}$
x bar	\bar{x}
x under	x

The diacritical mark is placed at the right height. The *bar* and *under* are made the right length for the entire construct, as in $\overline{x + y + z}$; other marks are centered.

Any input entirely within quotes ("...") is not subject to any of the font changes and spacing adjustments normally done by the equation setter. This provides a way to do your own spacing and adjusting if needed:

```
italic "sin(x)" + sin(x)
```

is

```
sin(x) + sin(x)
```

Quotes are also used to get braces and other EQN keywords printed:

"{ size alpha }"

is

```
{ size alpha }
```

and

roman "{ size alpha }"

is

{ size alpha }

The construction "" is often used as a place-holder when grammatically EQN needs something, but you don't actually want anything in your output. For example, to make ²He, you can't just type *sup* 2 roman He because a *sup* has to be a superscript on something. Thus you must say

"" sup 2 roman He

To get a literal quote use "\"". TROFF characters like (bs can appear unquoted, but more complicated things like horizontal and vertical motions with <math>h and v should always be quoted. (If you've never

Generally, *gsize* and *gfont* will appear at the beginning of a document but they can also appear thoughout a document: the global font and size can be changed as you will typically want the size of equations to match

[‡]Like this one, in which we have a few random expressions like x_i and π^2 . The sizes for these were set by the command *gsize* -2.

heard of h and v, ignore this section.)

Sometimes it's necessary to line up a series of equations at some horizontal position, often at an equals sign. This is done with two operations called *mark* and *lineup*.

The word *mark* may appear once at any place in an equation. It remembers the horizontal position where it appeared. Successive equations can contain one occurrence of the word *lineup*. The place where *lineup* appears is made to line up with the place marked by the previous *mark* if at all possible. Thus, for example, you can say

$$\neg EQ I$$

$$x+y mark = z$$

$$\neg EN$$

$$\neg EQ I$$

$$x lineup = 1$$

$$\neg EN$$

to produce

$$x + y = z$$
$$x = 1$$

For reasons too complicated to talk about, when you use EQN and '-ms', use either .EQ I or .EQ L. mark and *lineup* don't work with centered equations. Also bear in mind that *mark* doesn't look ahead;

x mark =1 ... x+y lineup =z

isn't going to work, because there isn't room for the x+y part after the *mark* remembers where the x is.

To get big brackets [], braces { }, parentheses (), and bars || around things, use the *left* and *right* commands:

```
left { a over b + 1 right }
~=~ left ( c over d right )
 + left [ e right ]
```

is

$$\left\{\frac{a}{b}+1\right\} = \left(\frac{c}{d}\right) + [e]$$

The resulting brackets are made big enough to cover whatever they enclose. Other characters can be used besides these, but the are not likely to look very good. One exception is the *floor* and *ceiling* characters:

left floor x over y right floor <= left ceiling a over b right ceiling

produces

$$\left\lfloor \frac{x}{y} \right\rfloor \leq \left\lceil \frac{a}{b} \right\rceil$$

Several warnings about brackets are in order. First, braces are typically bigger than brackets and parentheses, because they are made up of three, five, seven, etc., pieces, while brackets can be made up of two, three, etc. Second, big left and right parentheses often look poor, because the character set is poorly designed.

The *right* part may be omitted: a "left something" need not have a corresponding "right something". If the *right* part is omitted, put braces around the thing you want the left bracket to encompass. Otherwise, the resulting brackets may be too large.

If you want to omit the *left* part, things are more complicated, because technically you can't have a *right* without a corresponding *left*. Instead you have to say

for example. The *left* "" means a "left nothing". This satisfies the rules without hurting your output. There is a general facility for making vertical piles of things; it comes in several flavors. For example:

A ~=~ left [pile { a above b above c } ~ pile { x above y above z } right]

will make

$$A = \begin{bmatrix} a & x \\ b & y \\ c & z \end{bmatrix}$$

The elements of the pile (there can be as many as you want) are centered one above another, at the right height for most purposes. The keyword *above* is used to separate the pieces; braces are used around the entire list. The elements of a pile can be as complicated as needed, even containing more piles.

Three other forms of pile exist: *lpile* makes a pile with the elements left-justified; *rpile* makes a right-justified pile; and *cpile* makes a centered pile, just like *pile*. The vertical spacing between the pieces is somewhat larger for *l-*, *r-* and *cpiles* than it is for ordinary piles.

makes

sign(x) =
$$\begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$$

Notice the left brace without a matching right one.

- 12 -

It is also possible to make matrices. For example, to make a neat array like

```
\begin{array}{ccc} x_i & x^2 \\ y_i & y^2 \end{array}
```

you have to type

```
matrix {
    ccol { x sub i above y sub i }
    ccol { x sup 2 above y sup 2 }
}
```

This produces a matrix with two centered columns. The elements of the columns are then listed just as for a pile, each element separated by the word *above*. You can also use *lcol* or *rcol* to left or right adjust columns. Each column can be separately adjusted, and there can be as many columns as you like.

The reason for using a matrix instead of two adjacent piles, by the way, is that if the elements of the piles don't all have the same height, they won't line up properly. A matrix forces them to line up, because it looks at the entire structure before deciding what spacing to use.

A word of warning about matrices — *each column must have the same number of elements in it.* The world will end if you get this wrong.

In a mathematical document, it is necessary to follow mathematical conventions not just in display equations, but also in the body of the text, for example by making variable names like x italic. Although this could be done by surrounding the appropriate parts with .EQ and .EN, the continual repetition of .EQ and .EN is a nuisance. Furthermore, with '-ms', .EQ and .EN imply a displayed equation.

EQN provides a shorthand for short in-line expressions. You can define two characters to mark the left and right ends of an in-line equation, and then type expressions right in the middle of text lines. To set both the left and right characters to dollar signs, for example, add to the beginning of your document the three lines

.EQ delim .EN

Having done this, you can then say things like

Let alpha sub i be the primary variable, and let beta be zero. Then we can show that x sub 1 is >=0.

This works as you might expect — spaces, newlines, and so on are significant in the text, but not in the equation part itself. Multiple equations can occur in a single input line.

Enough room is left before and after a line that contains in-line expressions that something like

 $\sum_{i=1}^{n} x_i$ does not interfere with the lines surrounding

it.

To turn off the delimiters,

```
.EQ
delim off
.EN
```

Warning: don't use braces, tildes, circumflexes, or double quotes as delimiters — chaos will result. EQN provides a facility so you can give a frequently-used string of characters a name, and thereafter just type the name instead of the whole string. For example, if the sequence

x sub i sub 1 + y sub i sub 1

appears repeatedly throughout a paper, you can save re-typing it each time by defining it like this:

define xy 'x sub i sub 1 + y sub i sub 1'

This makes *xy* a shorthand for whatever characters occur between the single quotes in the definition. You can use any character instead of quote to mark the ends of the definition, so long as it doesn't appear inside the definition.

Now you can use *xy* like this:

```
\neg EQ \\ f(x) = xy \dots \\ \neg EN
```

and so on. Each occurrence of *xy* will expand into what it was defined as. Be careful to leave spaces or their equivalent around the name when you actually use it, so EQN will be able to identify it as special.

There are several things to watch out for. First, although definitions can use previous definitions, as in

```
.EQ
define xi´x sub i´
define xi1´xi sub 1´
.EN
```

don't define something in terms of itself' A favorite error is to say

```
define X ´ roman X ´
```

This is a guaranteed disaster, since X *is* now defined in terms of itself. If you say

define X ´roman "X" ´

however, the quotes protect the second X, and everything works fine.

EQN keywords can be redefined. You can make / mean *over* by saying

define / 'over'

or redefine over as / with

define over '/'

If you need different things to print on a terminal and on the typesetter, it is sometimes worth defining a symbol differently in NEQN and EQN. This can be done with *ndefine* and *tdefine*. A definition made with *ndefine* only takes effect if you are running NEQN; if you use *tdefine*, the definition only applies for EQN. Names defined with plain *define* apply to both EQN and NEQN.

Although EQN tries to get most things at the right place on the paper, it isn't perfect, and occasionally you will need to tune the output to make it just right. Small extra horizontal spaces can be obtained with tilde and circumflex. You can also say *back n* and *fwd n* to move small amounts horizontally. *n* is how far to move in 1/100's of an em (an em is about the width of the letter 'm'.) Thus *back 50* moves back about half the width of an m. Similarly you can move things up or down with *up n* and *down n*. As with *sub* or *sup*, the local motions affect the next thing in the input, and this can be something arbitrarily complicated if it is enclosed in braces.

Here is the complete source for the three display equations in the abstract of this guide.

.EQ I

G(z) mark = $e \sup \{ \ln G(z) \}$ $\tilde{=}$ exp left (sum from k>=1 {S sub k z sup k} over k right) $\tilde{=}$ prod from k>=1 e sup {S sub k z sup k/k} .EN .EO I lineup = left (1 + S sub 1 z + { S sub 1 sup 2 z sup 2 } over 2! + ... right) left (1+ { S sub 2 z sup 2 } over 2 + { S sub 2 sup 2 z sup 4 } over { 2 sup 2 cdot 2! } + ... right)EN .EQ I lineup = sum from m>=0 left (sum from pile { k sub 1 ,k sub 2 ,..., k sub m >=0 above $k \text{ sub } 1 + 2k \text{ sub } 2 + \dots + mk \text{ sub } m = m$ { S sub 1 sup {k sub 1} } over {1 sup k sub 1 k sub 1 ! } $\tilde{}$ { S sub 2 sup {k sub 2} } over {2 sup k sub 2 k sub 2 ! } ~

{ S sub m sup {k sub m} } over {m sup k sub m k sub m ! } right) z sup m .EN

If you don't use braces, EQN will do operations in the order shown in this list.

dyad vec under bar tilde hat dot dotdot fwd back down up fat roman italic bold size sub sup sqrt over from to

These operations group to the left:

over sqrt left right

All others group to the right.

Digits, parentheses, brackets, punctuation marks, and these mathematical words are converted to Roman font when encountered:

sin cos tan sinh cosh tanh arc max min lim log ln exp Re Im and if for det

These character sequences are recognized and translated as shown.

>=	≥
<=	\leq
==	=
!=	≠
+	±
->	\rightarrow
<-	\leftarrow
<<	~
>>	\gg
inf	$ \frac{\infty}{\partial} $
partial	9
half	$\frac{1}{2}$
prime	ĩ
approx	~
nothing	
cdot	
times	×
del	$\stackrel{ imes}{ abla} $
grad	∇
	•••
····,	,,
sum	Σ
int	ſ
	J
prod	Π
union	\cup
inter	\cap

To obtain Greek letters, simply spell them out in whatever case you want:

DELTA	Δ	iota	1
GAMMA	_ Г	kappa	ĸ
LAMBDA	Λ	lambda	λ
OMEGA	Ω	mu	μ
PHI	Φ	nu	v
PI	П	omega	ω
PSI	Ψ	omicron	0
SIGMA	Σ	phi	ϕ
THETA	Θ	pi	π
UPSILON	r	psi	ψ
XI	Ξ	rho	ρ
alpha	α	sigma	σ
beta	β	tau	τ
chi	X	theta	θ
delta	δ	upsilon	υ
epsilon	ε	xi	ξ
eta	η	zeta	ξ
gamma	γ		

These are all the words known to EQN (except for characters with names), together with the section where they are discussed.

above	17, 18	lpile	17
back	21	mark	15
bar	13	matrix	18
bold	12	ndefine	20
ccol	18	over	9
col	18	pile	17
cpile	17	rcol	18
define	20	right	16
delim	19	roman	12
dot	13	rpile	17
dotdot	13	size	12
down	21	sqrt	10
dyad	13	sub	7
fat	12	sup	7
font	12	tdefine	20
from	11	tilde	13
fwd	21	to	11
gfont	12	under	13
gsize	12	up	21
hat	13	vec	13
italic	12	~, ^	4,6
lcol	18	{ }	8
left	16	""	8, 14
lineup	15		

If you make a mistake in an equation, like leaving out a brace (very common) or having one too many (very common) or having a *sup* with nothing before it (common), EQN will tell you with the message

syntax error between lines x and y, file z

where x and y are approximately the lines between which the trouble occurred, and z is the name of the file in question. The line numbers are approximate — look nearby as well. There are also selfexplanatory messages that arise if you leave out a quote or try to run EQN on a non-existent file.

If you want to check a document before actually printing it (on UNIX only),

eqn files >/dev/null

will throw away the output but print the messages. If you use something like dollar signs as delimiters, it is easy to leave one out. This causes very strange troubles. The program *checkeq* (on GCOS, use *./checkeq* instead) checks for misplaced or missing dollar signs and similar troubles.

In-line equations can only be so big because of an internal buffer in TROFF. If you get a message "word overflow", you have exceeded this limit. If you print the equation as a displayed equation this message will usually go away. The message "line overflow" indicates you have exceeded an even bigger buffer. The only cure for this is to break the equation into two separate ones.

On a related topic, EQN does not break equations by itself — you must split long equations up across multiple lines by yourself, marking each by a separate .EQEN sequence. EQN does warn about equations that are too long to fit on one line.

To print a document that contains mathematics on the UNIX typesetter,

eqn files | troff

If there are any TROFF options, they go after the TROFF part of the command. For example,

eqn files | troff -ms

To run the same document on the GCOS typesetter, use

eqn files | troff –g (other options) | gcat

A compatible version of EQN can be used on devices like teletypes and DASI and GSI terminals which have half-line forward and reverse capabilities. To print equations on a Model 37 teletype, for example, use

neqn files | nroff

The language for equations recognized by NEQN is identical to that of EQN, although of course the output is more restricted.

To use a GSI or DASI terminal as the output device,

neqn files | nroff -Tx

where x is the terminal type you are using, such as 300 or 300S.

EQN and NEQN can be used with the TBL program[2] for setting tables that contain mathematics. Use TBL before [N]EQN, like this:

tbl files | eqn | troff tbl files | neqn | nroff

We are deeply indebted to J. F. Ossanna, the author of TROFF, for his willingness to extend TROFF to make our task easier, and for his continuous assistance during the development and evolution of EQN. We are also grateful to A. V. Aho for advice on language design, to S. C. Johnson for assistance with the YACC compiler-compiler, and to all the EQN users who have made helpful suggestions and criticisms.

References

[1]J. F. Ossanna, "NROFF/TROFF User's Manual", Bell Laboratories Computing Science Technical Report #54, 1976.

[2]M. E. Lesk, "Typing Documents on UNIX", Bell Laboratories, 1976.

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[3]M. E. Lesk, "TBL — A Program for Setting Tables", Bell Laboratories Computing Science Technical Report #49, 1976.

- 15 -

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